due to the neutralization of the space charge by positive ions is evidence of ionization. When a hot mercury arc was used no increase in thermionic current was observed.

The process of ionization which we have here demonstrated seems to be of a very interesting type, and it is proposed to continue the study of it with the idea of working out the details of the cumulative action involved.

[1] Steubing, W., *Physik. Zeits.*, **10**, 1909 (787–793).

[2] Kingdon, K. H., *Physic. Rev.*, **21**, 1923 (408–418).

MODIFIED AND UNMODIFIED SCATTERING COEFFICIENTS OF X-RAYS IN MATTER

BY O. K. DEFOE AND G. E. M. JAUNCEY

WASHINGTON UNIVERSITY, ST. LOUIS, MO.

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In all methods of calculating the scattering coefficient from the intensity of the X-rays scattered in a certain direction ϕ and the intensity of the primary rays the assumption has been made that the scattered rays have the same wave-length and therefore the same absorption coefficient as the primary rays. However A. H. Compton[1] has shown that part of the scattered rays are scattered with a change of wave-length

$$\delta\lambda = h \text{ vers } \phi/mc \tag{1}$$

where ϕ is the angle of scattering and h, m and c have their usual significance. Compton calls the X-rays scattered with change of wave-length and therefore of absorption coefficient modified rays, while he calls the rays scattered without change of wave-length unmodified rays. Due to the existence of both modified and unmodified rays it becomes necessary to recogonize two scattering coefficients. We shall represent the linear scattering coefficient of the modified X-rays scattered per unit solid angle in the direction ϕ with the forward direction of the primary rays by s' and the coefficient of the unmodified rays by s.

METHOD I. In Crowther's method[2] of experimentally obtaining the scattering coefficient in a direction ϕ a thin slab of the scattering substance is placed with its faces vertical at the axis of an X-ray spectrometer. The slab is turned so that the normal to either face bisects the scattering angle ϕ. The ionization chamber is first placed at a scattering angle ϕ and the saturated ionization current I_ϕ measured. Next the chamber is placed at zero angle so as to receive the fraction of the primary X-rays which penetrate through the slab and the current I measured, the position of the slab remaining the same as when I_ϕ is measured. We now proceed to develop formulas for the calculation of both s and s'. The technique of our method is to transfer a certain thickness of aluminum from a position where this thickness intercepts the primary beam of X-rays before they fall upon the scattering slab to a position between the scattering slab and the ionization chamber which is set at an angle ϕ. Different readings of I_ϕ are obtained for the two positions of the aluminum. Let t be the thickness of the scattering slab, P the thickness of the aluminum in the primary rays, T the thickness of aluminum in the scattered rays, and R the distance from the scattering slab to the window of the ionization chamber. In our method $P + T$ is kept constant. Also let μ_1 be the linear absorption coefficient of the primary X-rays and μ_2 that of the modified rays in aluminum, let μ_3 be the absorption coefficient of the primary and μ_4 that of the modified rays in the scattering substance, and let μ_5 be that of the primary rays in air and μ_6 that of the modified rays in air. (There is air between the ionization chamber window and the scattering slab.) Also let A be the area of the ionization chamber window (A must be large enough to admit the entire pencil of the primary X-rays whose intensity is I). We now have

$$I_\phi = C\{S + S'e^{-(kT+b)}\,(e^g-1)/g\} \tag{2}$$

where

$$\left.\begin{aligned} C &= AIt/R^2 \cos \tfrac{1}{2}\phi \\ k &= (\mu_2-\mu_1) \\ b &= (\mu_6-\mu_5)\,R + g \\ g &= (\mu_4-\mu_3)t/\cos \tfrac{1}{2}\phi \end{aligned}\right\} \tag{2A}$$

Experimentally the magnitude of t is such that g is small and in this case Eq. (2) reduces to

$$I_\phi = C\{S + S'e^{-(kT+b)}\}. \tag{3}$$

If now we take two thicknesses d_1 and d_2 of T($P + T$ is kept constant) and measure the corresponding values i_1 and i_2 of I_ϕ and also if we measure I, then

$$S = \frac{i_2 - i_1 e^{-kd}}{C(1-e^{-kd})} \tag{4}$$

$$S' = \frac{i_1 - i_2}{Ce^{-(kd_1+b)}\,(1-e^{-kd})} \tag{5}$$

where $d = (d_2-d_1)$ is the thickness of the aluminum transferred from the primary rays to the scattered rays. In the measurement of I the thickness of aluminum passed through by the primary rays is $P + T$. It should be noted that T and therefore d_1 and d_2 include the thickness of the aluminum window of the ionization chamber. In Eqs. (4) and (5) all the quantities in the right-hand members are experimental magnitudes with the exceptions of k and b. We now use Compton's formula, Eq. (1) to determine $\delta\lambda$ for a given angle ϕ. By reference to curves of absorption coefficients plotted against wave-lengths we can determine both k and b which depend on the change of the absorption coefficient of the modified rays. Also it is seen that the ratio s'/s may be determined without a knowledge of C and therefore of I.

METHOD II. The scattering coefficients, s and s', can also be measured by scattering from the incident face of a thick block. In this case we take $t = \infty$. The block is set so that its face bisects the scattering angle ϕ. In this method the intensity I_0 of the primary X-rays penetrating through the air and a thickness $P + T$ of aluminum and entering the ionization chamber when the chamber is set at zero angle is measured. Under these conditions

$$I_\phi = D\{S + BS'e^{-kT}\} \tag{6}$$

where

$$\left.\begin{aligned} D &= AI_0/2R^2\mu_3 \\ B &= \frac{e^{-(\mu_6-\mu_5)R}}{1 + (\mu_4-\mu_3)/2\mu_3} \end{aligned}\right\} \tag{6A}$$

and

If we obtain experimentally two values i_1 and i_2 of I_ϕ for values d_1 and d_2 of the thickness T, we have

$$S = \frac{i_2 - i_1 e^{-kd}}{D(1-e^{-kd})} \tag{7}$$

and

$$S = \frac{i_1 - i_2}{DB\,(1-e^{-kd})} \tag{8}$$

where again $d = d_2 - d_1$.

The ratio s'/s can evidently be determined without a knowledge of D and therefore of I_0.

THE MODIFIED AND UNMODIFIED SCATTERING COEFFICIENTS OF COPPER

In a preliminary experiment X-rays of effective wave-length 0.40 angstrom (determined by half value absorption in aluminum) were scattered by copper at $\phi = 90°$. The ratio s'/s was determined by Method II, while the value of I_ϕ for a single thickness of T was determined by Method I, the value of I also being determined. This enabled us to calculate both s and s'. The results are shown in table 1.

TABLE 1

λ ANGSTROM	s'/s	s CM^{-1}	s' CM^{-1}	s_0 CM^{-1}	s/s_0	s'/s_0
0.40	3.8	0.025	0.095	0.097	0.26	0.98

In the fifth column of table 1 is given the value of s at $\phi = 90°$ for X-rays scattered by copper calculated on J. J. Thomson's theory of scattering.[3]

We might call $s + s'$ the total scattering coefficient whence we have $(s + s')/s_0 = 1.24$.

Jauncey's theory of the unmodified line[4] gives a method of calculating the ratio of the energy in the modified to that in the unmodified line when the K, L, M, etc., critical absorption wave-lengths of the scattering substance are known. In the present case the theoretical value of the proportion of the scattered energy in the modified to the total scattered energy is 83.5 per cent, whereas the experimental value is 79 per cent. Too much confidence, however, must not be placed in this agreement as our experimental errors were not as small as we would have desired. In conclusion it should be remarked that A. H. Compton[5] used the method of transferring sheets of aluminum from the primary to the scattered rays to determine the change of wave-length of the modified rays. Compton, however, did not use the method to determine s and s'.

[1] A. H. Compton, *Physic Rev.*, **22,** 408 (1923).

[2] J. A. Crowther, *Proc. Roy. Soc.*, **86,** 478 (1912).

[3] J. J. Thomson, Conduction of Electricity through Gases, 2nd. Ed., p. 325.

[4] G. E. M. Jauncey, *Physic Rev.*, **25,** 314 (1925) and **25,** 723 (1925).

[5] A. H. Compton, *Phil. Mag.*, **46,** 897 (1923).

THE ENERGY REAPPEARING AS CHARACTERISTIC X-RAYS WHEN X-RAYS ARE ABSORBED IN COPPER

BY G. E. M. JAUNCEY AND O. K. DEFOE

WASHINGTON UNIVERSITY, ST. LOUIS, MO.

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In the previous paper by DeFoe and Jauncey[1] appearing in this issue of these PROCEEDINGS, two methods have been described for obtaining both the modified and unmodified scattering coefficients of X-rays in matter. The same methods may be used for distinguishing and obtaining the fluorescent coefficient in addition to the two scattering coefficients. For instance, when X-rays fall upon copper part of the X-rays are scattered and part are absorbed. Of the absorbed rays part of the energy reappears as K, L, M, etc., characteristic X-rays of copper. We introduce the linear fluorescent coefficient per unit solid angle in a direction ϕ and represent it by f. The fluorescent rays have a different wave-length from the primary exciting rays and the problem is mathematically the same as that for separating the modified and unmodified coefficients, represented by s' and s respectively. Equation (3) of the previous paper becomes

$$I_\phi = C\{S + S'e^{-(kT + b)} + fe^{-(k_1T + b_1)}\} \qquad (1)$$